HALE SCHOOL

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2

Solutions

Section One
Booklet 1 of 3
(Calculator-free)

59

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	**9**	**9**	**50**	**59**	**35**
Section Two: Calculator-assumed	12	12	100	88	65
				Total	100

Instructions to candidates

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2. Write your answers in this Question/Answer booklet.

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4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Question 1 **(6 marks)**

(a) Evaluate $\frac{a^3}{b^{0.5}}$ when $a = 2 \times 10^2$ and $b = 4 \times 10^4$, writing your answer without the use of scientific notation. (3 marks)

Solution
$\frac{a^3}{b^{0.5}} = \frac{8 \times 10^6}{\sqrt{4} \times 10^2}$ $= \frac{8}{2} \times 10^4$ $= 40\,000$
Specific behaviours
✓ simplifies numerator ✓ simplifies denominator ✓ simplifies

(b) Determine the value of x when $49^x = 7\sqrt{7}$. (3 marks)

Solution
$7^{2x} = 7^1 \times 7^{\frac{1}{2}}$ $= 7^{\frac{3}{2}}$ $2x = 3 \Rightarrow x = \frac{3}{4}$
Specific behaviours
✓ write LHS and RHS as powers of 7 ✓ equates indices ✓ solves

Question 2 **(5 marks)**

(a) Expand $(x+1)^4$. (2 marks)

Solution
$(x+1)^4 = (1)(x)^4(1)^0 + (4)(x)^3(1)^1 + (6)(x)^2(1)^2 + (4)(x)^1(1)^3 + (1)(x)^0(1)^4$ $= x^4 + 4x^3 + 6x^2 + 4x + 1$
Specific behaviours
✓ correct powers ✓ correct coefficients

(b) Hence determine the gradient of the curve $y = (x+1)^4$ at the point $(-2, 1)$. (3 marks)

Solution
$\frac{dy}{dx} = 4x^3 + 12x^2 + 12x + 4$ $x = -2 \Rightarrow \frac{dy}{dx} = 4(-8) + 12(4) + 12(-2) + 4 = -4$
Specific behaviours
✓ differentiates expression from (a) ✓ substitutes $x = -2$ ✓ evaluates gradient

Question 3 **(9 marks)**

(a) Simplify

(i) $\frac{d}{dx}(\sqrt[3]{x} - \frac{1}{x^2})$ (2 marks)

Solution
$\frac{1}{3x^{\frac{2}{3}}} + \frac{2}{x^3}$
Specific behaviours
✓ correct derivative of 1st term ✓ correct derivative of 2nd term (accept negative indices

(ii) $\lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$ (1 mark)

Solution
$4x^3$
Specific behaviours
✓ correct derivative

(b) Determine the equation of the tangent to the curve $y = x^3 - 4x + 3$ when $x = -2$. (4 marks)

Solution
$\frac{dy}{dx} = 3x^2 - 4$ $x = -2, \quad y = -8 + 8 + 3 = 3, \quad \frac{dy}{dx} = 12 - 4 = 8$ $y - 3 = 8(x + 2) \Rightarrow y = 8x + 19$
Specific behaviours
✓ correct derivative ✓ calculates y-coordinate ✓ calculates gradient ✓ correct equation of tangent, in any form

(c) Determine $f(x)$ given $f'(x) = 4x - 5$ and $f(2) = -3$. (2 marks)

Solution
$f(x) = 2x^2 - 5x + c$ $2(2)^2 - 5(2) + c = -3 \Rightarrow c = -1$ $f(x) = 2x^2 - 5x - 1$
Specific behaviours
✓ correct antiderivative with constant ✓ correct $f(x)$

Question 4 **(8 marks)**

Solve each equation below for x:

(a) $\dfrac{3x}{x-5} = \dfrac{2}{3}$ (2 marks)

Solution
$9x = 2x - 10$ $7x = -10$ $x = -\dfrac{10}{7}$
Specific behaviours
✓ cross multiplies ✓ solves

(b) $(x+3)(x-3) = 8x$ (3 marks)

Solution
$x^2 - 9 = 8x$ $x^2 - 8x - 9 = 0$ $(x+1)(x-9) = 0$ $x = -1, \quad x = 9$
Specific behaviours
✓ expands and equates to zero ✓ factorises ✓ solves for 2 solutions

(c) $\sqrt{2}\sin x + 1 = 0,\ 0° \leq x \leq 360°$ (3 marks)

Solution
$\sin x = -\dfrac{1}{\sqrt{2}}$ $x = 225°, \quad x = 315°$
Specific behaviours
✓ rearranges equation ✓ one correct solution ✓ both correct solutions

Question 5 **(4 marks)**

A cubic is given by $f(x) = x^3 - x^2 - 24x - 36$.

(a) Show that the cubic has a root when $x = -2$. (1 mark)

Solution
$x = -2, \quad y = -8 - 4 + 48 - 36 = 48 - 48 = 0$
Specific behaviours
✓ substitutes and obtains zero

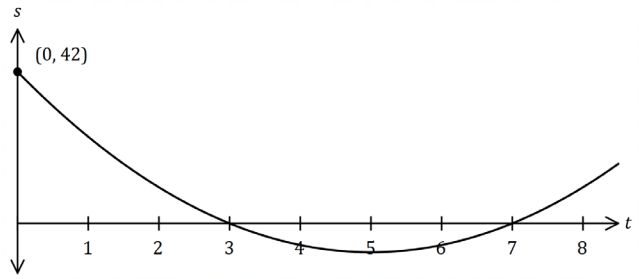
(b) Determine the coordinates of the other two roots of the cubic. (3 marks)

Solution
$x^3 - x^2 - 24x - 36 = (x + 2)(x^2 + ax - 18)$ $= (x + 2)(x^2 - 3x - 18)$ $= (x + 2)(x + 3)(x - 6)$ Other roots at $(-3, 0)$ and $(6, 0)$.
Specific behaviours
✓ obtains quadratic factor by inspection ✓ factorises quadratic ✓ states both roots as coordinates

Question 6 **(6 marks)**

A small body moves in a straight line so that its displacement s metres from a fixed point O after t seconds is given by $s = at^2 + bt + c$.

The position-time graph of the body is shown below.



(a) Determine the values of the constants a, b and c. (3 marks)

Solution
$d = a(t-3)(t-7)$ $42 = a(-3)(-7) \Rightarrow a = 2$ $d = 2(t^2 - 10t + 21)$ $= 2t^2 - 20t + 42$ $a = 2, \quad b = -20, \quad c = 42$
Specific behaviours
✓ states $c = 42$ ✓ appropriate method of substitution to calculate another value ✓ states third value

(b) Determine the displacement of the body when its velocity is 12 ms^{-1}. (3 marks)

Solution
$v = 4t - 20$ $4t - 20 = 12 \Rightarrow t = 8$ $d(8) = 2(8-3)(8-7) = 10$ m
Specific behaviours
✓ equation for velocity ✓ solves for time ✓ substitutes for displacement

Question 7 **(7 marks)**

The first three terms, in order, of a sequence are $x + 4, x$ and $2x - 15$.

Determine the value of the fourth term of the sequence if:

(a) the sequence is arithmetic (3 marks)

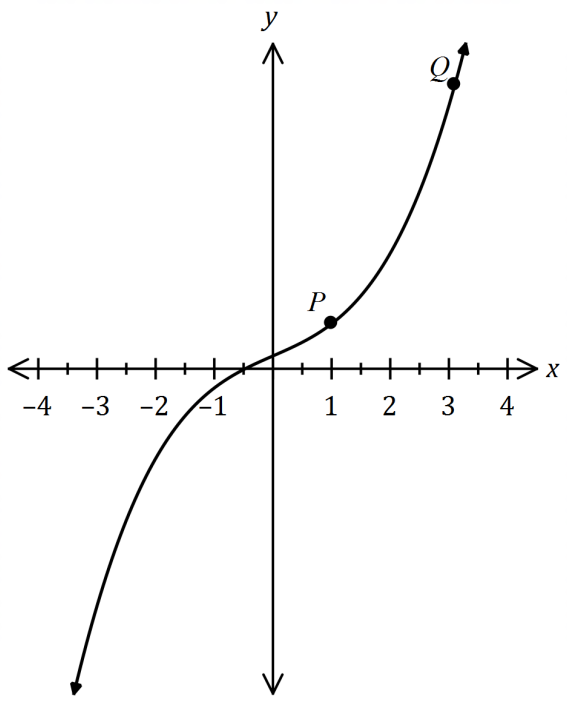
Solution
$d = (x) - (x + 4) = (2x - 15) - (x)$ $-4 = x - 15 \Rightarrow x = 11$ $d = -4$ $T_1 = 15\,, \quad T_2 = 11, \quad T_3 = 7$ $T_4 = 3$
Specific behaviours
✓ equates differences ✓ solves for x ✓ finds T_4

(b) the sequence is geometric (4 marks)

Solution
$r = \dfrac{x}{x + 4} = \dfrac{2x - 15}{x}$ $x^2 = (2x - 15)(x + 4)$ $x^2 = 2x^2 + 8x - 15x - 60$ $0 = x^2 - 7x - 60 = (x - 12)(x + 5)$ $x = 12, x = -5$ If $x = 12$, $r = \frac{3}{4}$ and $T_1 = 16\,,\ T_2 = 12,\ T_3 = 9$ and $\quad T_4 = 6.75$ If $x = -5$, $r = 5$ and $T_1 = -11,\ T_2 = -5,\ T_3 = -25$ and $\quad T_4 = -125$
Specific behaviours
✓ equates ratios ✓ equates quadratic to 0 ✓ solves for x ✓ finds value(s) of T_4

Question 8 **(7 marks)**

Let $f(x) = x^3 + 4x + 2$. The graph of $y = f(x)$ is shown below.



(a) Points P and Q lie on the curve with x-coordinates 1 and 3 respectively.

(i) Determine $f(1)$ and $f(3)$. (1 mark)

Solution
$f(1) = 7, \quad f(3) = 41$
Specific behaviours
✓ both values correct

(ii) Determine the gradient of a straight line through P and Q. (2 marks)

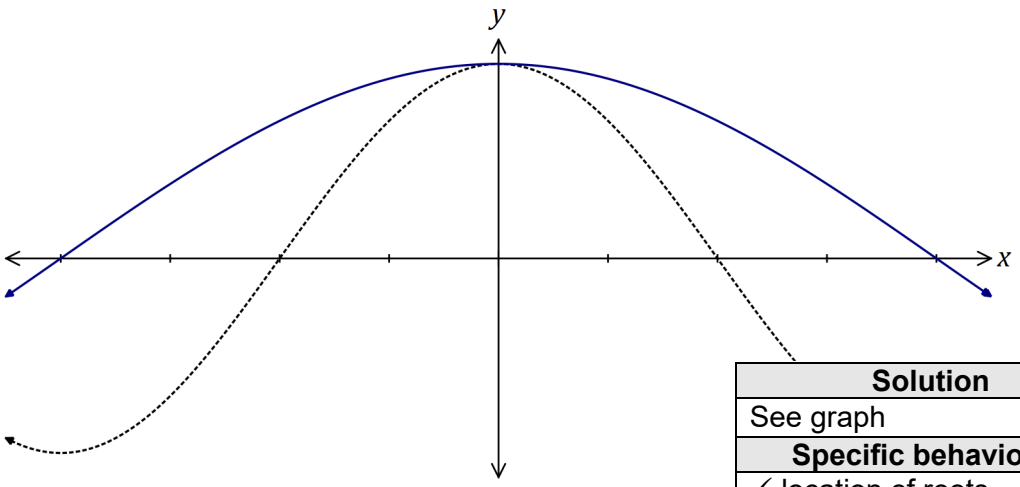
Solution
$m = \frac{41 - 7}{2} = 17$
Specific behaviours
✓ substitutes correctly into gradient formula ✓ simplifies

(b) Find $f'(x)$ using the calculus method of first principles. (4 marks)

Solution
$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \dfrac{(x+h)^3+4(x+h)+2-x^3-4x-2}{h}$ $= \lim_{h\to 0} \dfrac{x^3+3hx^2+3h^2x+h^3+4x+4h+2-x^3-4x-2}{h}$ $= \lim_{h\to 0} \dfrac{h(3x^2+3hx+3h^2+4}{h}$ $= 3x^2+4$
Specific behaviours
✓ states first principles equation ✓ substitutes $f(x)$ and $f(x+h)$ ✓ factorises h ✓ evaluates limit

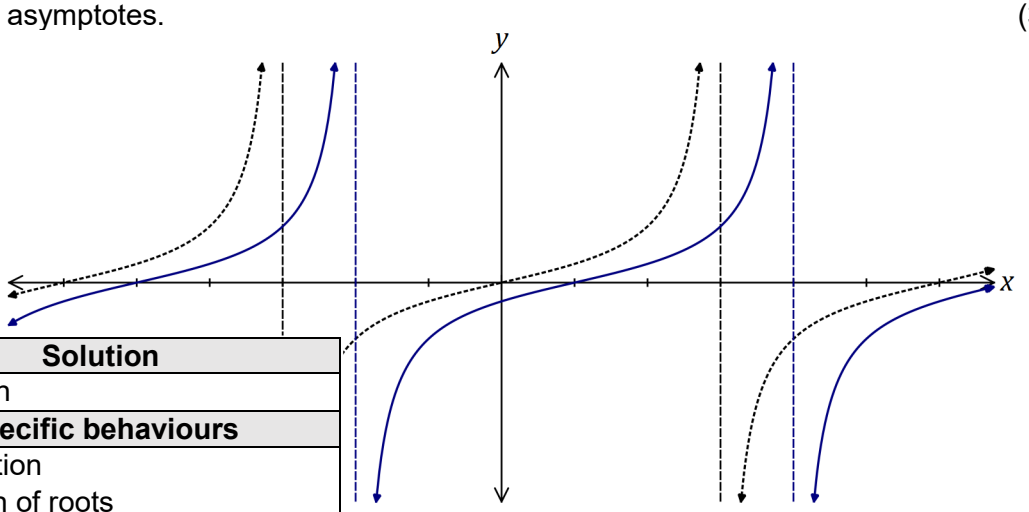
Question 9 **(7 marks)**

(a) The graph of $y = \cos x$ is shown below. On the same axes, sketch $y = \cos\left(\frac{1}{2}x\right)$. (2 marks)



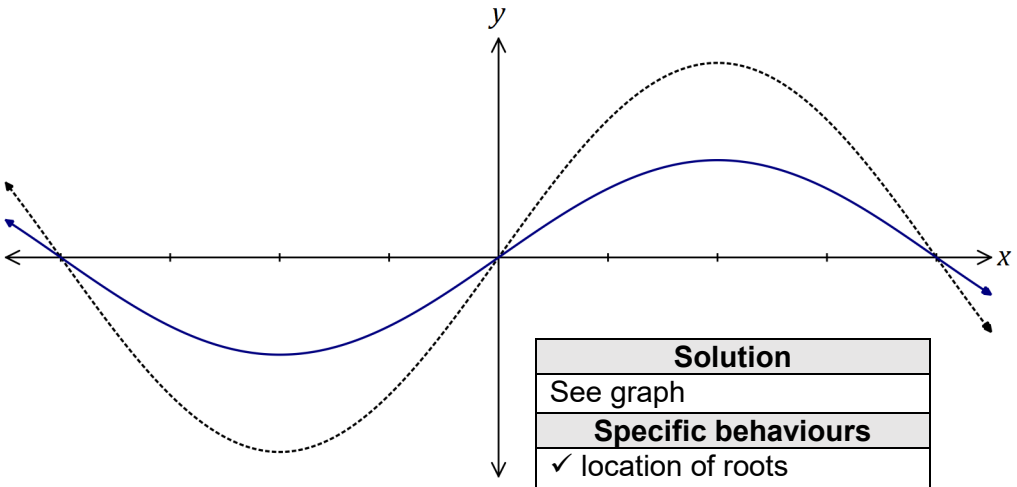
Solution
See graph
Specific behaviours
✓ location of roots
✓ correct amplitude/shape

(b) The graph of $y = \tan x$ is shown below. On the same axes, sketch $y = \tan\left(x - \frac{\pi}{6}\right)$ and all its asymptotes. (3 marks)



Solution
See graph
Specific behaviours
✓ translation
✓ location of roots
✓ asymptotes

(c) The graph of $y = 2\sin x$ is shown below. On the same axes, sketch $y = \cos\left(x - \frac{\pi}{2}\right)$. (2 marks)



Solution
See graph
Specific behaviours
✓ location of roots
✓ correct amplitude/shape

HALE SCHOOL

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2

SOLUTIONS

Section Two
Booklet 2 of 3
(Calculator-assumed)

45

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
TWO Question/Answer Booklets – complete BOTH
Formula Sheet – retained from Section One

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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Question 10 **(6 marks)**

(a) A sequence is defined by $T_{n+1} = T_n - 1.8,\ T_1 = 975$. Determine:

(i) the value of T_{120} (1 mark)

Solution
$T_{120} = 760.8$
Specific behaviours
✓ correct value

(ii) the sum of the first 120 terms of this sequence (2 marks)

Solution
$S_{120} = \frac{120}{2} \times (975 + 760.8)$ $= 104\ 148$
Specific behaviours
✓ uses sum formula ✓ correct sum (CAS only OK for both)

(b) Another sequence is defined by $A_n = 975(0.2)^{n-1}$. Determine:

(i) the value of A_5 (1 mark)

Solution
$A_5 = 1.56$
Specific behaviours
✓ correct value

(ii) the value S_n approaches as $n \to \infty$ (2 marks)

Solution
$S_\infty = \frac{975}{1 - 0.2}$ $= 1\ 218.75$
Specific behaviours
✓ uses sum to infinity formula ✓ correct value (CAS only OK for both) * if students give value as 1218.8, 1 out of 2 marks

Question 11 **(7 marks)**

The temperature T (measured in $°C$) of a cast taken out of an oven cools according to the model $T = 960(0.93)^t$, where t is the time in minutes since the cast was removed from the oven.

(a) Determine the fall in temperature of the cast during the first 3 minutes. (2 marks)

Solution
$T = 960(0.93)^3 \approx 772°C$ $\Delta T = 960 - 772 = 188°C$
Specific behaviours
✓ value of T when $t = 3$ ✓ correct drop

(b) State the name of this type of function. (1 mark)

Solution
Exponential.
Specific behaviours
✓ correct name

(c) The temperature of the cast falls to a room temperature of $14°C$.

(i) Determine the time taken, to the nearest second, for the cast to reach room temperature. (2 marks)

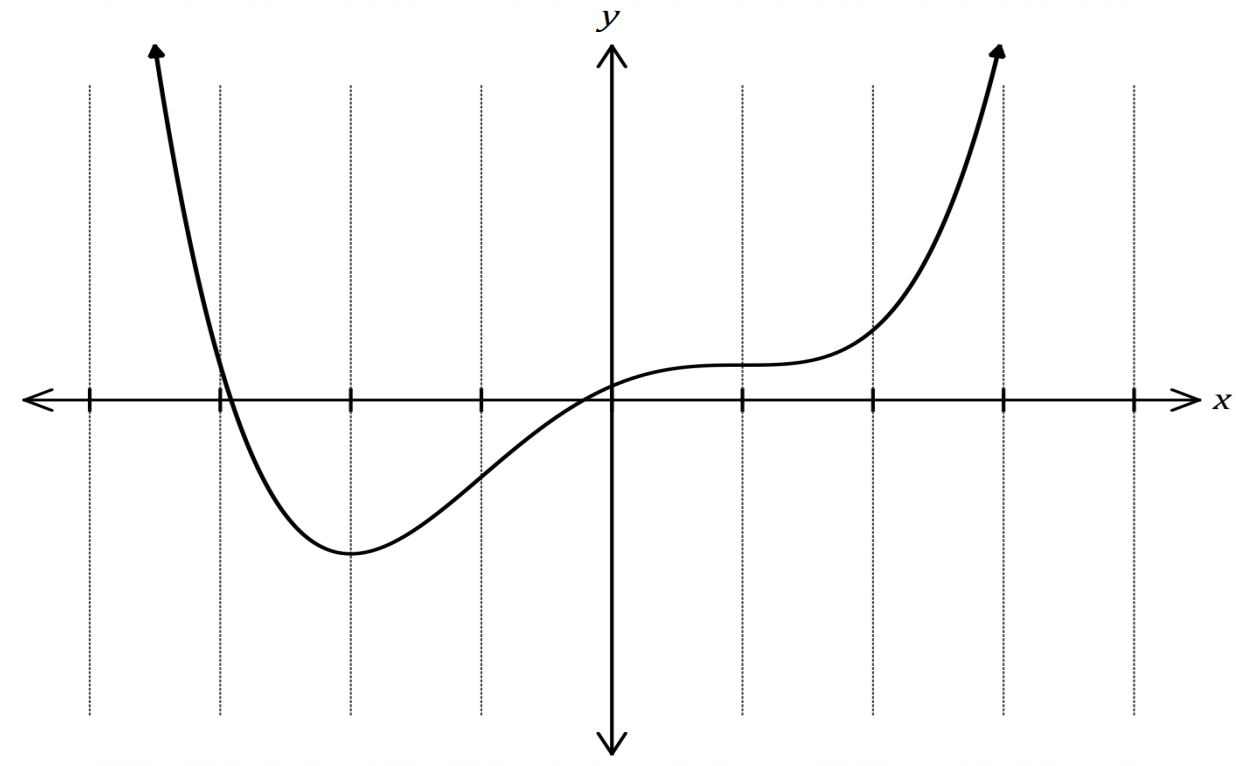
Solution
$960(0.93)^t = 14 \Rightarrow t = 58 \text{ minutes and } 16 \text{ seconds}$
Specific behaviours
✓ solves equation to find time ✓ correct time in seconds

(ii) Comment on the usefulness of the model for large values of t. (2 marks)

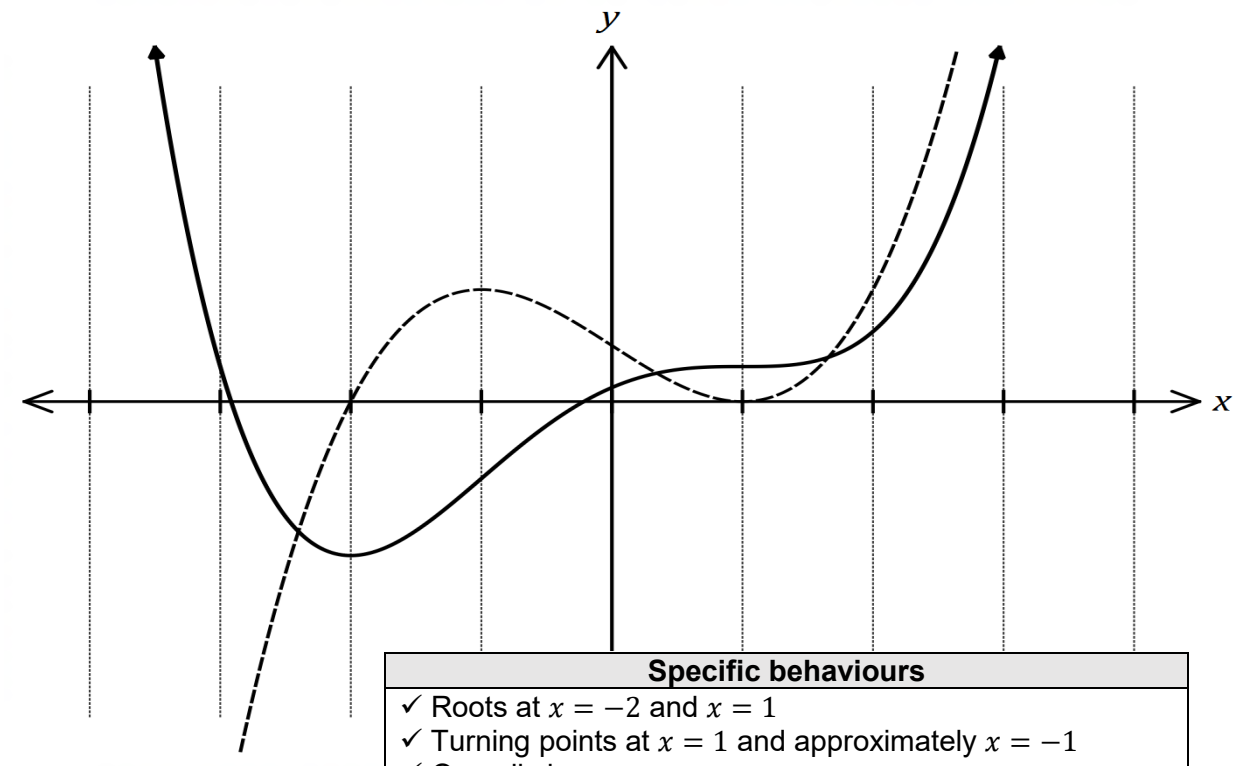
Solution
For large values of t the model shows that $T \to 0$ but the temperature of the cast only falls to $14°C$ and so model not valid for large T.
Specific behaviours
✓ states not valid ✓ states reason

Question 12 **(3 marks)**

The graph of $f(x)$ is shown below:



On the axes below, draw the graph of $f'(x)$.



Specific behaviours
✓ Roots at $x = -2$ and $x = 1$
✓ Turning points at $x = 1$ and approximately $x = -1$
✓ Overall shape

Question 13 **(8 marks)**

(a) Calculate the area of the minor segment that subtends an arc of $108°$ in a circle of diameter 130 cm. (2 marks)

Solution
$108° = \frac{3\pi}{5}, \qquad r = \frac{130}{2} = 65$ $A = \frac{1}{2}(65)^2\left(\frac{3\pi}{5} - \sin\frac{3\pi}{5}\right) \approx 1\,972.86 \text{ cm}^2$
Specific behaviours
✓ converts angle, uses correct radius ✓ calculates area

(b) A chord of length 56 cm subtends an angle of $\frac{\pi}{7}$ at the centre of a circle. Calculate the radius of the circle. (2 marks)

Solution
$56 = 2r\sin\left(\frac{1}{2} \times \frac{\pi}{7}\right)$ $r \approx 125.83 \text{ cm}$
Specific behaviours
✓ substitutes into formula ✓ calculates radius

(c) Parallelogram $PQRS$ has side $QR = 24$ cm, side $RS = 39$ cm and an area of 460 cm^2. Determine the lengths of the diagonals of $PQRS$. (4 marks)

Solution
$\frac{1}{2}(24)(39)\sin x = \frac{460}{2}$ $x = 29.44°, 150.56°$ $L_1 = \sqrt{24^2 + 39^2 - 2(24)(39)\cos 29.44}$ ≈ 21.6 cm $L_2 = \sqrt{24^2 + 39^2 - 2(24)(39)\cos 150.56}$ ≈ 61.1 cm
Specific behaviours
✓ equation for half area ✓ both angles of parallelogram ✓ correct length of one diagonal ✓ second correct length

Question 14 **(7 marks)**

A function is defined by $f(x) = \frac{x^4}{6} - \frac{2x^3}{3}$

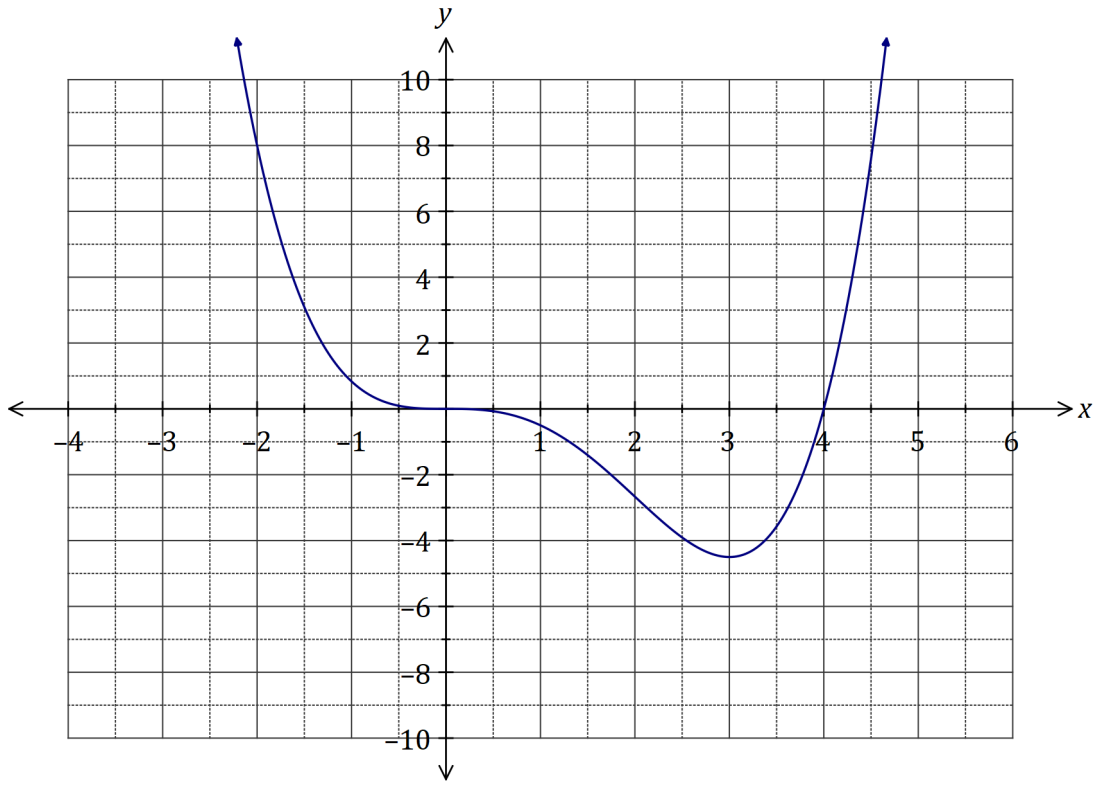
(a) Use calculus to determine the coordinates of all stationary points of the function. (3 marks)

Solution
$f'(x) = \frac{2}{3}x^3 - 2x^2$ $\frac{2}{3}x^3 - 2x^2 = 0 \Rightarrow x = 0, x = 3$ $f(0) = 0, \quad f(3) = -4.5$ Stationary points at $(0, 0)$ and $(3, -4.5)$
Specific behaviours
✓ correct derivative ✓ correct zeros of derivative ✓ correct coordinates

(b) Use calculus to determine the nature of the stationary points found in (a). (2 marks)

Solution
$f''(x) = 2x^2 - 4x$ $f''(0) = 0$ therefore horizontal point of inflection $f''(3) = 6$ threrefore min Stationary points at $(0, 0)$ and $(3, -4.5)$
Specific behaviours
✓ correct use of 2nd derivative test or sign test ✓ correct conclusion based off testing

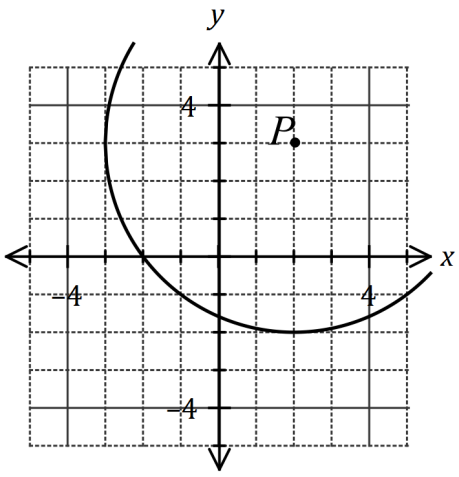
(c) Sketch the graph of $y = f(x)$ on the axes below. Clearly indicate key features. (2 marks)



Solution
See graph
Specific behaviours
✓ key points: HPI at (0, 0), minimum at $(3, -4.5)$, root at $(4, 0)$
✓ correct smooth curvature

Question 15 **(6 marks)**

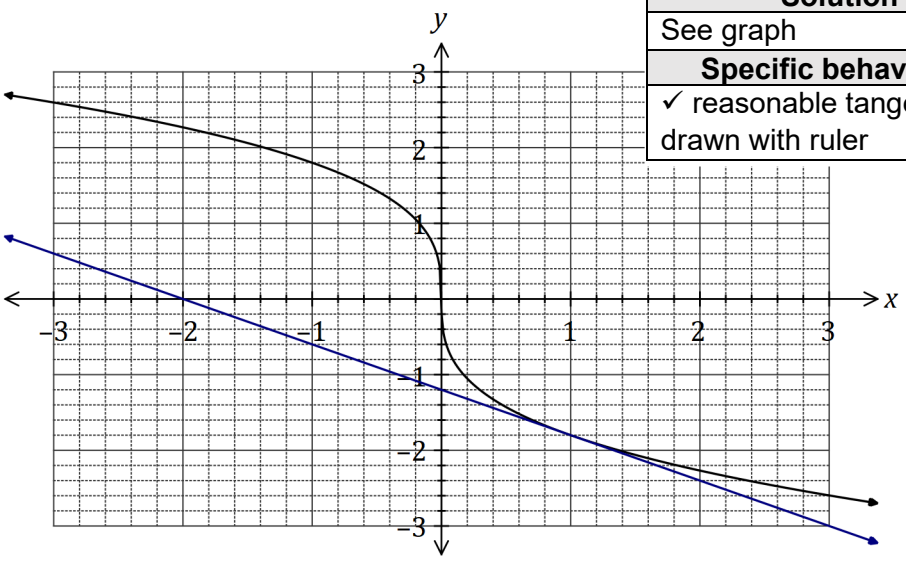
(a) Part of the circle $x^2 + y^2 = ax + by + c$ is shown below. The point P marks the centre of the circle. Determine the values of the constants a, b and c. (4 marks)



Method 1
$(x-2)^2 + (y-3)^2 = 5^2$
$x^2 + y^2 = 4x + 6y + 12$
$a = 4, \quad b = 6, \quad c = 12$
Specific behaviours
✓ circle in factored form
✓ correct radius and centre
✓ expands into required form
✓ correct values of a, b and c

Method 2
$(-3)^2 + 3^2 = -3a + 3b + c$
$2^2 + (-2)^2 = 2a - 2b + c$
$5^2 + (-1)^2 = 5a - b + c$
$a = 4, \quad b = 6, \quad c = 12$
Specific behaviours
✓ equation 1
✓ equation 2
✓ equation 3
✓ correct values of a, b and c

(b) The graph of a power function $y = g(x)$ is shown below:



Solution
See graph
Specific behaviours
✓ reasonable tangent drawn with ruler

(i) Draw the tangent to the curve when $x = 1$. (1 mark)

(ii) Use your tangent line to estimate $g'(1)$. (1 mark)

Solution
$g'(1) \approx -\frac{1.2}{2} \approx -0.6$
Specific behaviours
✓ gradient matches tangent drawn above, must be simplified

Question 16 **(8 marks)**

A mobile phone retailer classified recent sales of 625 phones by the age of customer and if the phone was bought outright or on a plan. A summary of the data is shown in the table below.

	Aged under 30	Aged 30 or over	Total
Bought outright	108	p	232
Bought on a plan	q	152	r
Total			625

(a) Determine the values of p, q and r shown in the table. (3 marks)

Solution
$p = 232 - 108 = 124$ $q = 393 - 152 = 241$ $r = 625 - 232 = 393$
Specific behaviours
✓ correct p value ✓ correct q value ✓ correct r value

(b) A recent sale is selected at random from those recorded above. Event A occurs if the customer was aged under 30 and event B occurs if the phone was bought outright. Determine the following probabilities:

(i) $P(\bar{B})$ (1 mark)

Solution
$P(\bar{B}) = \frac{393}{625}\ (= 0.6288)$
Specific behaviours
✓ correct probability

(ii) $P(B \cup A)$ (1 mark)

Solution
$P(B \cup A) = \frac{625 - 152}{625} = \frac{473}{625}\ (= 0.7568)$
Specific behaviours
✓ correct probability

(iii) $P(\bar{A} \cap B)$ (1 mark)

Solution
$P(\bar{A} \cap B) = \frac{124}{625}\ (= 0.1984)$
Specific behaviours
✓ correct probability

(iv) $P(A\,|\bar{B}\,)$ (2 marks)

Solution
$P(A\,\|\bar{B}\,) = \frac{241}{393}\ (\approx 0.613)$
Specific behaviours
✓ numerator ✓ denominator

HALE SCHOOL

Semester Two Examination, 2018

Question/Answer booklet

SOLUTIONS

MATHEMATICS METHODS UNITS 1 AND 2

Section Two
Booklet 3 of 3
(Calculator-assumed)

43

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7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Question 17 **(8 marks)**

Two water containers, initially empty, are being filled with water. The amount of water added to container A each minute follows a geometric sequence, with 3 mL poured in during the first minute and 3.6 mL poured in during the second minute. The amount of water added to container B each minute follows an arithmetic sequence, with 5 mL poured in during the first minute and 10 mL poured in during the second minute.

(a) The amount of water poured into container A during the n^{th} minute, A_n, is given by $a(r)^{n-1}$. State the value of the constants a and r. (2 marks)

Solution
$a = 3$ $r = \frac{3.6}{3} = 1.2$
Specific behaviours
✓ value of a ✓ value of r

(b) Determine the total amount of water in container A at the end of the 12^{th} minute. (2 marks)

Solution
$S_{12} = \frac{3(1 - 1.2^{12})}{1 - 1.2}$ $= 118.74 \text{ mL}$
Specific behaviours
✓ uses sum formula ✓ correct amount (CAS only OK)

(c) How long does it take to fill container B with 330 mL of water? (2 marks)

Solution
$\frac{n}{2}\big(2(5) + (n-1)(5)\big) = 330$ $n = 11 \text{ minutes}$
Specific behaviours
✓ uses sum formula ✓ correct time (CAS only OK)

(d) Container A first holds more water than container B at the end of minute m.

(i) Determine the value of m. (1 mark)

Solution
$m = 27$
Specific behaviours
✓ correct value

(ii) State, to the nearest mL, how much more water A contains than B at this time. (1 mark)

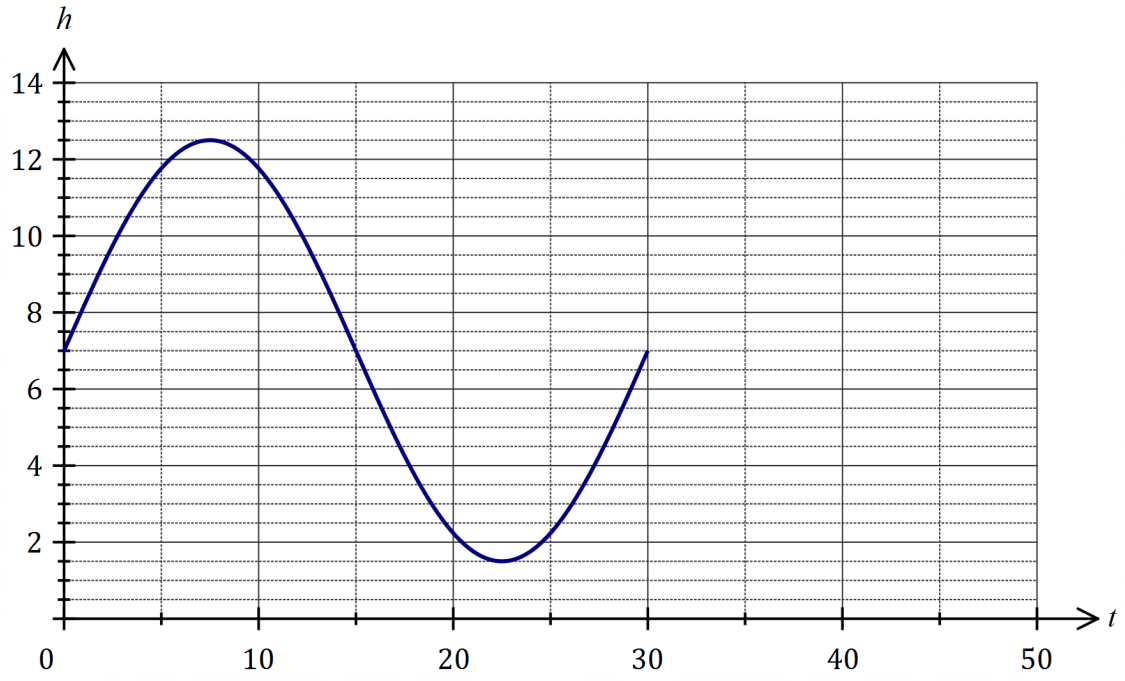
Solution
$2046 - 1890 = 156 \text{ mL}$
Specific behaviours
✓ correct value

Question 18 **(9 marks)**

The height, h metres, above level ground of a seat on a steadily rotating Ferris wheel t seconds after observations began was given by

$$h = 5.5 \sin\left(\frac{\pi t}{15}\right) + 7$$

(a) Draw the graph of the height of the seat against time, for one revolution of the Ferris wheel, on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ correct y-intercept ✓ correct turning points ✓ smooth curve over 1 revolution

(b) State the domain and range of your graph in part (a). (2 marks)

Solution
Domain: $0 \leq t \leq 30$ Range: $1.5 \leq h \leq 12.5$
Specific behaviours
✓ correct domain using correct notation ✓ correct range using correct notation

(c) Explain why this is a graph of a function rather than a relation. (1 mark)

Solution
It is one-to-one, passes vertical line test
Specific behaviours
✓ mentions being one-to-one or passing vertical line test

(d) At what time did the seat first reach a height of 5 metres? (1 mark)

Solution
$t = 16.78\ \text{s}$
Specific behaviours
✓ correct time

(e) Determine the change in height of the seat between $t = 5$ and $t = 6$, giving your answer rounded to the nearest cm. (2 marks)

Solution
$h(6) = 12.230811$ $h(5) = 11.76314$ $\delta h = 0.467671 = 0.47\ \text{m}$
Specific behaviours
✓ determines both heights ✓ states difference to nearest cm

Question 19 **(10 marks)**

A pyramid with a rectangular base of length l and width w has perpendicular height h. The length of the base is three times its width. The sum of the width, length and height is 180 cm.

(a) Calculate the length, height and volume of the pyramid when $w = 15$ cm. (2 marks)

Solution
$l = 3 \times 15 = 45, \quad h = 180 - 15 - 45 = 120$ $V = \frac{1}{3}(15 \times 45) \times 120 = 27\,000 \text{ cm}^3$
Specific behaviours
✓ correct length and height ✓ correct volume

(b) Show that the volume of the pyramid is given by $V = 180w^2 - 4w^3$. (2 marks)

Solution
$l = 3w, \quad h = 180 - w - 3w = 180 - 4w$ $V = \frac{1}{3}(w \times 3w)(180 - 4w)$ $= 180w^2 - 4w^3$
Specific behaviours
✓ expressions for length and height ✓ substitutes width, length and height correctly

(c) Use calculus to determine the maximum volume of the pyramid and state the dimensions required to achieve this volume. (6 marks)

Solution
$$\frac{dV}{dw} = 360w - 12w^2$$ $$360w - 12w^2 = 0 \Rightarrow w = 0, 30$$ $$V_{max} = 180(30)^2 - 4(30)^3 = 54\,000 \text{ cm}^3$$ $$\frac{d^2V}{dw^2} = 360 - 24(30) = -360 \; therefore \; max$$ $$w = 30 \text{ cm}, \quad l = 90 \text{ cm}, \quad h = 60 \text{ cm}$$
Specific behaviours
✓ correct derivative using given variables ✓ set derivative equal zero ✓ solves ✓ correct maximum volume ✓ justify that that point is max ✓ correct dimensions (must have units)

Question 20 **(9 marks)**

A council took a random sample of 125 and 172 properties from suburbs P and Q respectively. A total of 36 of the properties in the sample were in arrears with their rates, and 21 of these properties were in suburb Q. 'In arrears' means that payment of rates is overdue.

(a) Council officers wanted to choose 4 of the properties that were in arrears. How many different selections of properties are possible? (2 marks)

Solution
$\binom{36}{4} = 58\,905$
Specific behaviours
✓ indicates use of nCr formula
✓ correct number

(b) Determine the probability that one randomly chosen property from the sample

(i) is not in arrears and is in suburb Q. (2 marks)

Solution
$P = \dfrac{172-21}{125+172} = \dfrac{151}{297}\ (\approx 0.508)$
Specific behaviours
✓ numerator
✓ denominator

(ii) is in suburb P given that it is in arrears. (2 marks)

Solution
$P = \dfrac{36-21}{36} = \dfrac{15}{36} = \dfrac{5}{12}\ (= 0.41\overline{6})$
Specific behaviours
✓ numerator
✓ denominator

(c) Justifying your answer with independence testing, comment on whether being in arrears with rates is independent of the suburb the property is in.

(3 marks)

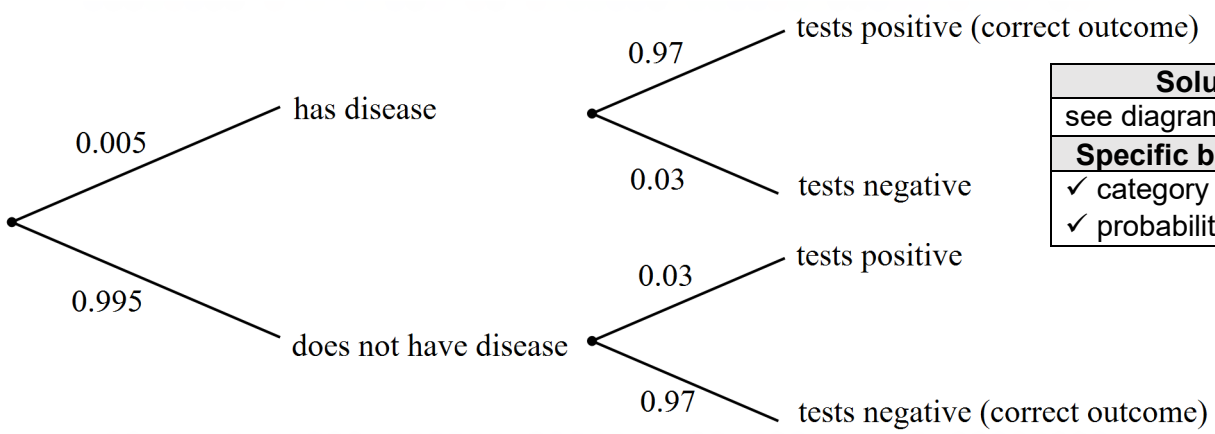
Method 1
$P(\text{Arrears}\|P) = \dfrac{15}{125} = 12.0\%$
$P(\text{Arrears}\|\text{Q}) = \dfrac{21}{172} \approx 12.2\%$
Hence being in arrears is independent of suburb, as conditional probabilities are very similar.
Specific behaviours
✓ calculates $P(\text{Arrears}\|P)$
✓ calculates $P(\text{Arrears}\|Q)$
✓ conclusion based off evidence

Method 2
If independent, $P(A) \cap P(Q) = P(A) \times P(Q)$
$\dfrac{21}{297} \approx \dfrac{36}{297} \times \dfrac{172}{297} \approx 0.07$
Hence being in arrears is independent of suburb.
Specific behaviours
✓ uses correct independence test
✓ calculates LHS and RHS
✓ conclusion based off evidence

Question 21 **(7 marks)**

A diagnostic test for a disease has a 97% chance of giving the correct outcome and it is known that 0.5% of all sheep on a station have the disease. It can be assumed that the correct outcome of the test is independent of whether a sheep has the disease.

(a) Draw a probability tree diagram to show the information given above. (2 marks)



Solution
see diagram
Specific behaviours
✓ category labels
✓ probability labels

(b) A sheep is randomly selected for the test from those on the station. Determine the probability that the sheep actually has the disease if the test indicates that it does. (4 marks)

Solution
$P(D \cap T) = 0.005 \times 0.97 = 0.00485$ $P(\overline{D} \cap T) = 0.995 \times 0.03 = 0.02985$ $P(T) = 0.00485 + 0.02985 = 0.0347$ $P(D\|T) = \dfrac{0.00485}{0.0347} = \dfrac{97}{694} \approx 0.140$
Specific behaviours
✓ calculates $P(D \cap T)$
✓ calculates $P(\overline{D} \cap T)$
✓ indicates $P(T)$
✓ correct probability

(c) Comment on how effective the diagnostic test is. (1 mark)

Solution
not effective: only 14% of sheep actually have the disease if the test indicates they do
Specific behaviours
✓ correct conclusion with reasoning based on parts (a) or (b)